

# ***An Introduction to Formal Argumentation***

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# ***An Example (1/2) [Prakken]***

Paul: My car is very safe.

Olga: Why?

Paul: Since it has an airbag.

Olga: It is true that your car has an airbag, but I do not think that this makes your car safe, because airbags are unreliable: the newspapers had several reports on cases where airbags did not work.

Paul: I also read that report but a recent scientific study showed that cars with airbags are safer than cars without airbags, and scientific studies are more important than newspaper reports.

Olga: OK, I admit that your argument is stronger than mine. However, your car is not very safe, since its maximum speed is much too high.

# *Arguments and attacks*

Argument: expresses one or more reasons that lead to a proposition

$a, b, c \Rightarrow d$       or       $a, b \Rightarrow c; c \Rightarrow d$

An argument can *attack* another argument  
rebutting attack:

attack one of the *conclusions* of the other argument:

$e, f, g \Rightarrow \neg d$       against       $a, b, c \Rightarrow d$

undercutting attack:

attack the *reasons* of the other argument

$e, f, g, \Rightarrow [a, b, c \not\Rightarrow d]$  against       $a, b, c \Rightarrow d$

## ***Example (2/2)***

A: My car is very safe, since it has an airbag:  
 $\text{has\_airbag} \Rightarrow \text{safe}$

B: The newspapers say that airbags are not reliable, so having an airbag is not a good reason why your car is safe  
 $\text{say}(\text{npr}, \neg \text{rel}(\text{airbag})) \Rightarrow \neg \text{rel}(\text{airbag})$   
 $\neg \text{rel}(\text{airbag}) \Rightarrow [\text{has\_airbag} \not\Rightarrow \text{safe}]$

C: Scientific reports say that airbags are reliable.  
 $\text{say}(\text{sr}, \text{rel}(\text{airbag})) \Rightarrow \text{rel}(\text{airbag})$

# *How Arguments Interact (1/2)*

●  
A

A: my car is very safe since it has an airbag

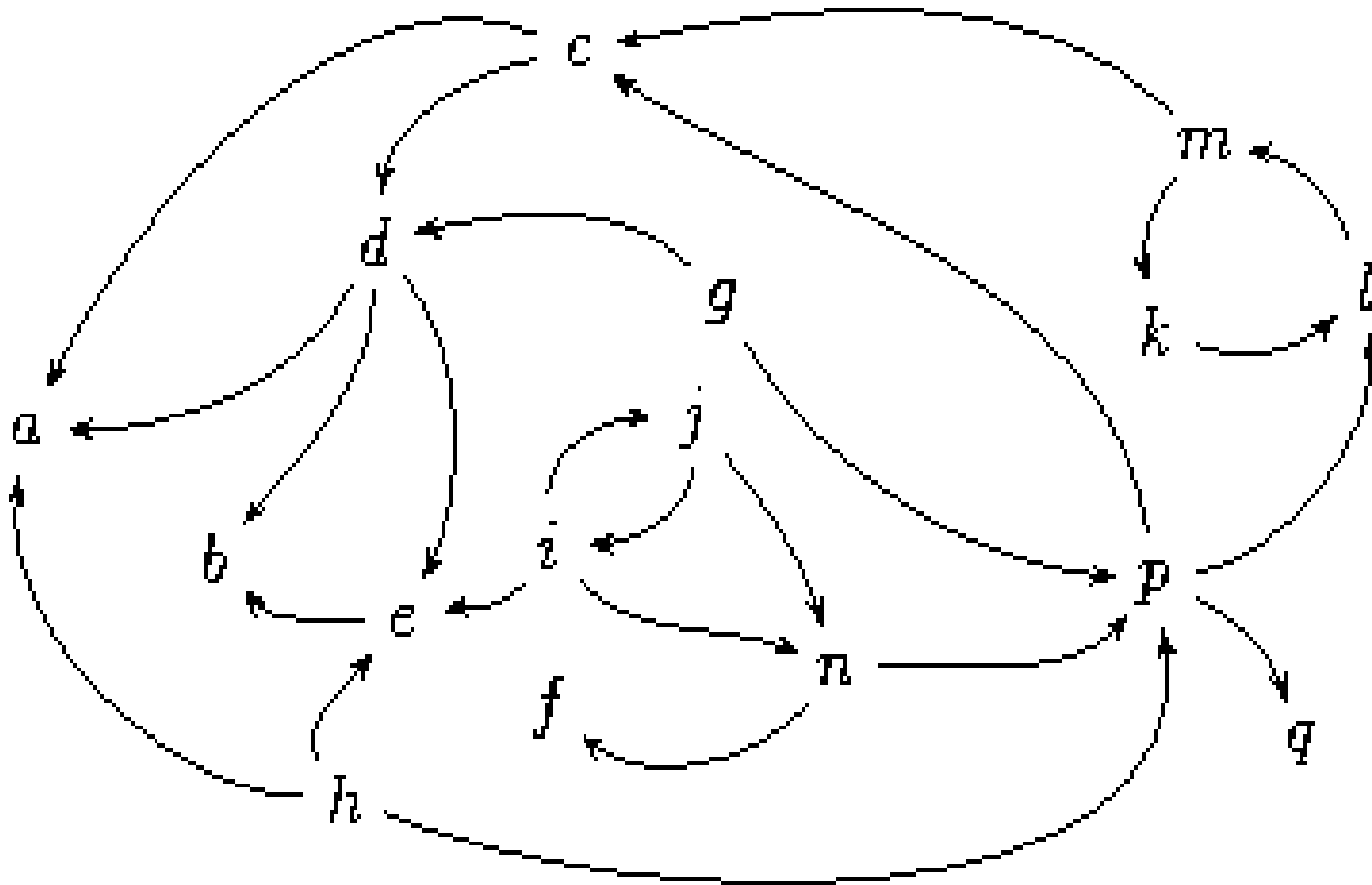
● ← ●  
A B

B: newspapers say that airbags are unreliable

● ← ● ← ●  
A B C

C: scientific reports say that airbags are reliable, and these are more important than newspapers

# ***How Arguments Interact (2/2)***



# ***Argumentation: what is it good for?***

Legal reasoning: CATO/HYPO  
use argumentation tools for supporting lawyers

Medical reasoning: CRUK/CARREL  
helping doctors to suggest the best  
treatment for their patients

# ***Nonmonotonic Logic***

$$\Phi \vdash \varphi$$
$$\Rightarrow$$
$$\Phi \cup \Psi \vdash \varphi$$



# ***The Argumentation Approach***

generate arguments based on  
a knowledge base  
see how these arguments defeat each  
other  
determine which arguments can  
be seen as justified  
take the conclusions of the justified  
arguments

# ***Argumentation in Agent Systems***

For internal reasoning of single agents  
reasoning about beliefs, goals, intentions etc is often  
defeasible

For interaction between multiple agents  
information exchange involves explanation  
collaboration and negotiation involve conflict of opinion and persuasion

# ***What Arguments Look Like (1/2)***

## ***Arguments as Sets of Assumptions***

Given a knowledge base  $(K, \text{Ass})$

Argument:  $(A, c)$  with  $A \subseteq \text{Ass}$  s.t.:

$$A \cup K \models c$$

$$A \cup K \not\models \perp$$

$$\nexists a \in A: A \setminus \{a\} \cup K \models c$$

(Besnard & Hunter, 2001)

# ***What Attacks Look Like (1/2)***

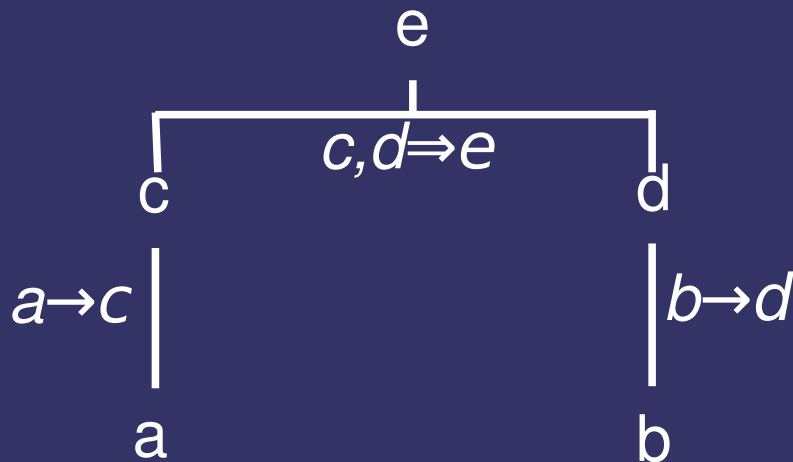
## ***Arguments as Sets of Assumptions***

Assumption attack:

$(A_2, c_2)$  attacks  $(A_1, c_1)$  iff  $\neg c_2 \in A_1$

# ***What Arguments Look Like (1/2)***

## ***Arguments as Trees Constructed with Rules***

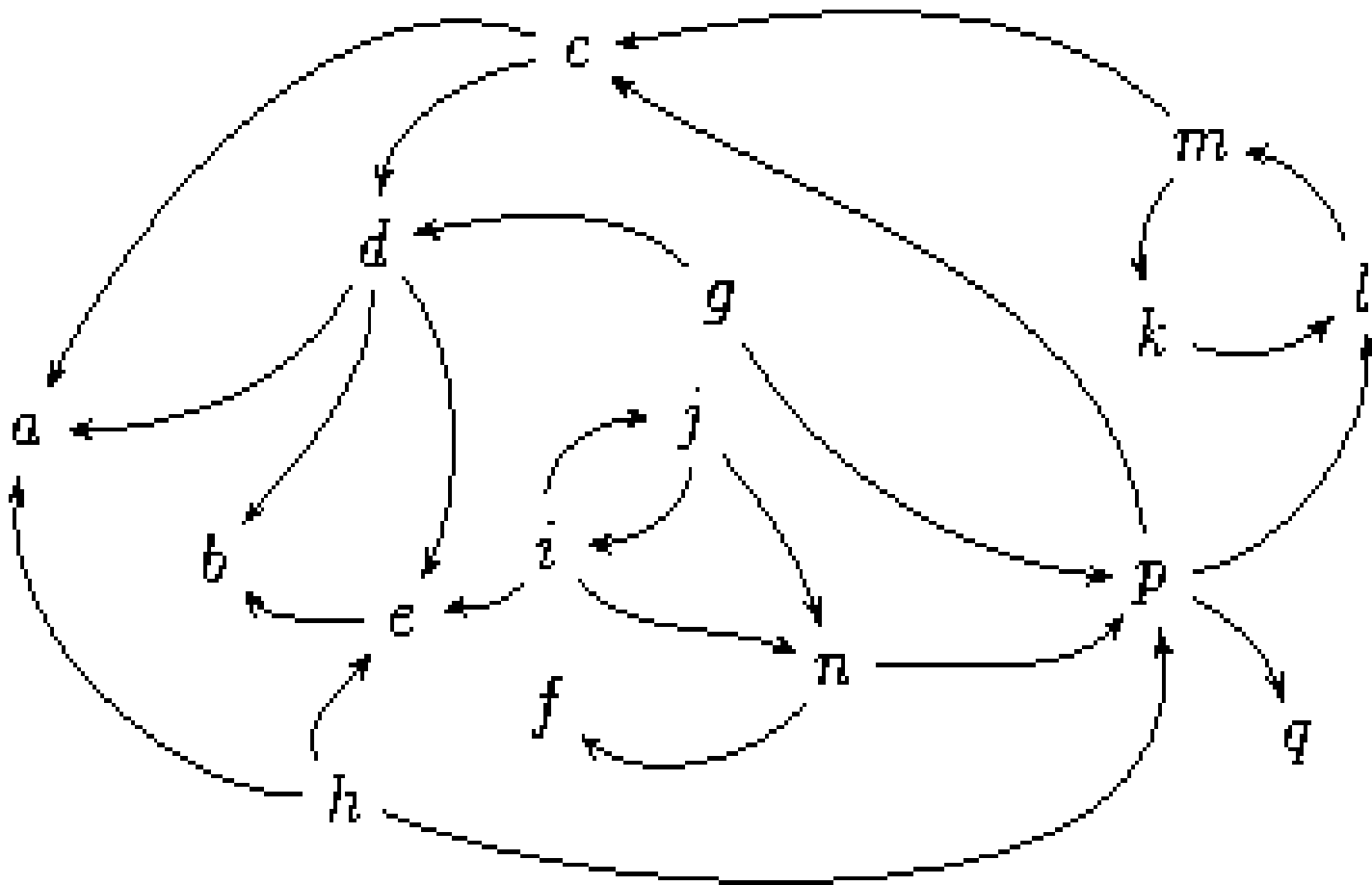


$$((a) \rightarrow c), ((b) \rightarrow d) \Rightarrow e$$

strict rule ( $\rightarrow$ ): “from ... it always follows that...”

defeasible rule ( $\Rightarrow$ ): “from ... it usually follows that...”

# ***How Arguments Interact (2/2)***



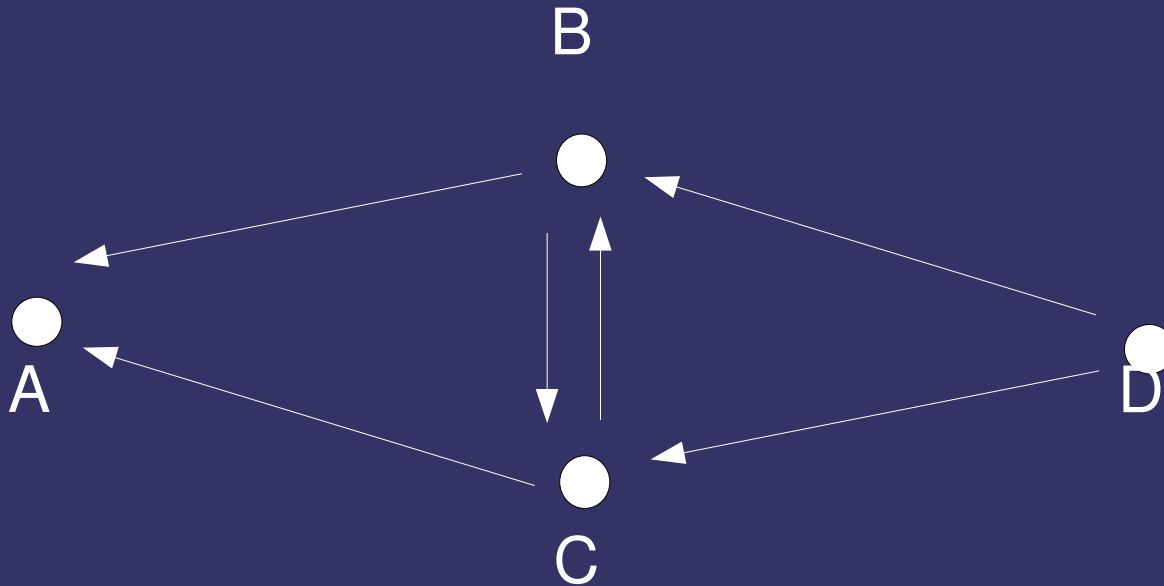
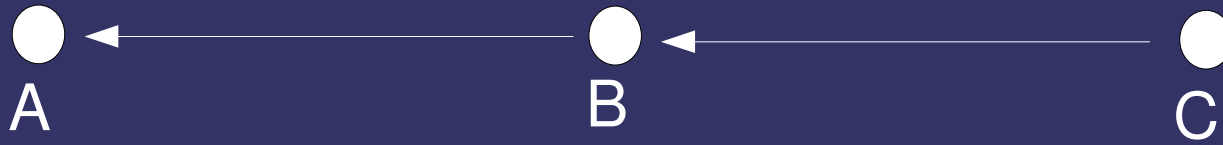
# ***Argument Evaluation Postulate***

argument labels: in, out, undec

An argument is in  
iff all its defeaters are out

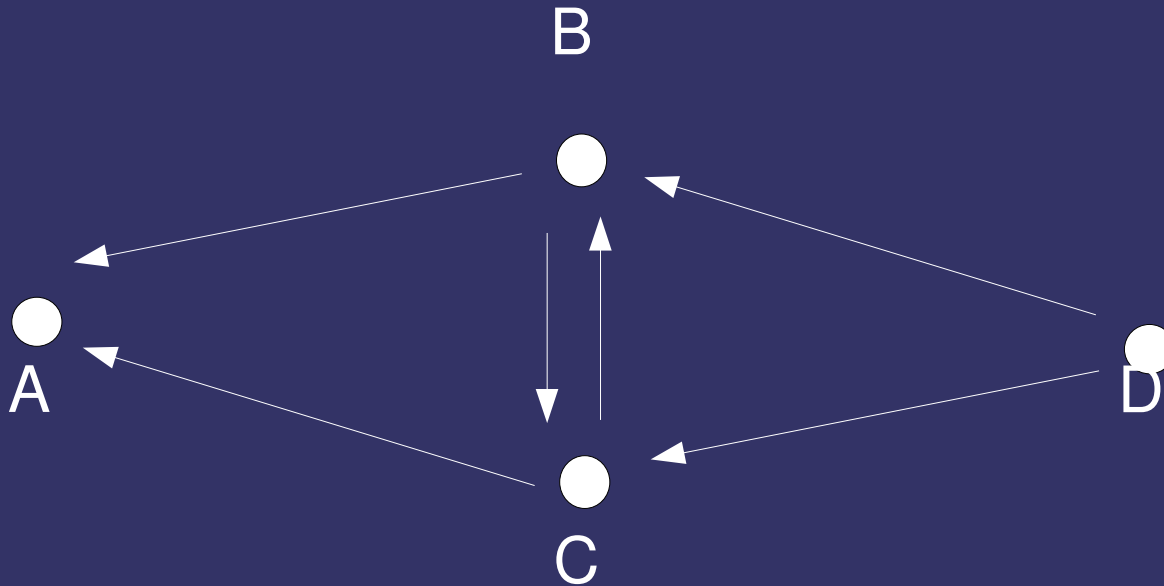
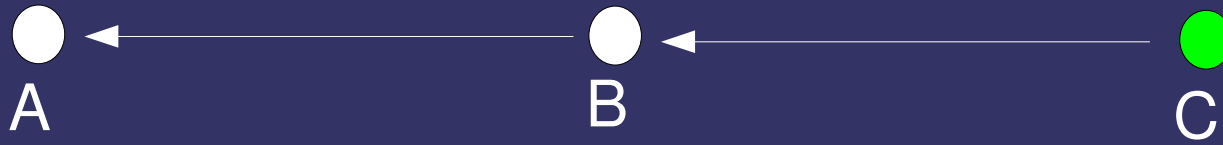
An argument is out  
iff it has a defeater that is in

# ***Applying the Evaluation Postulate (1/3)***

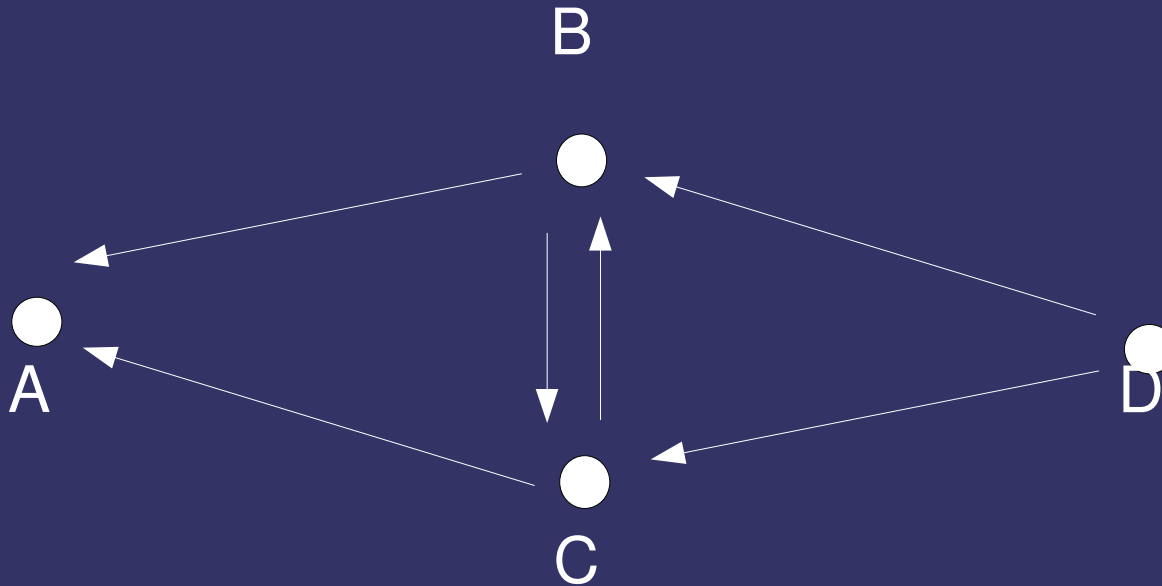
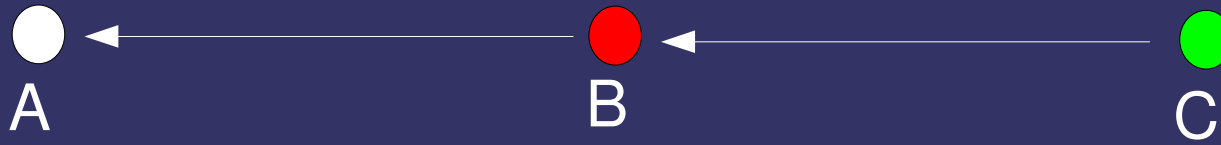




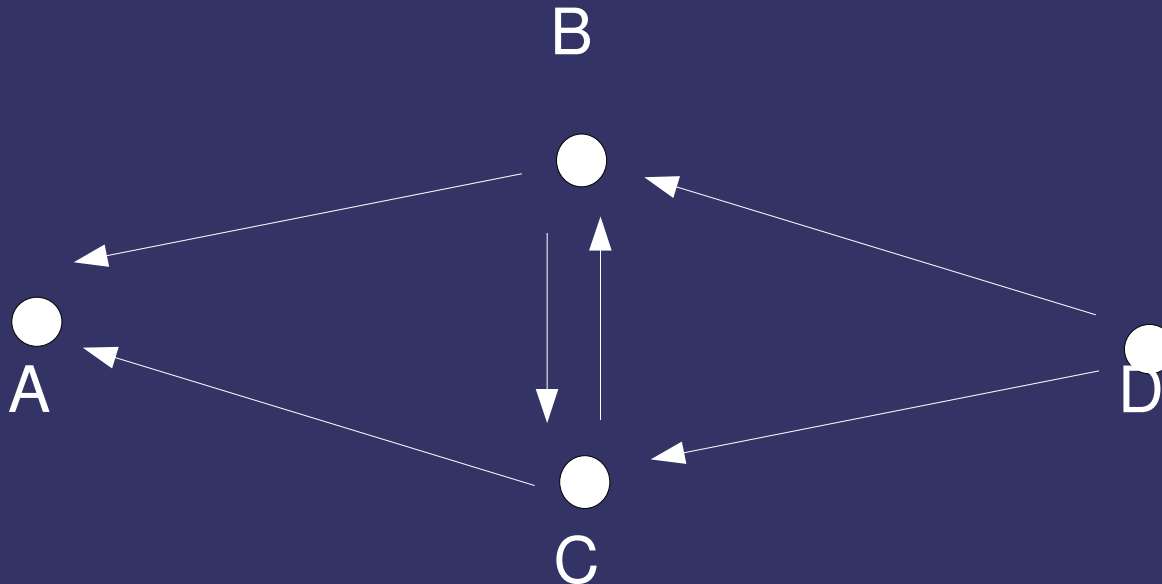
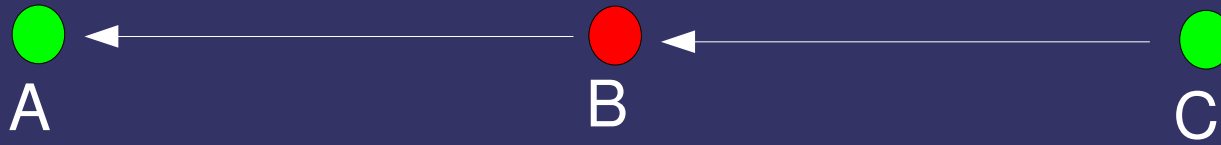
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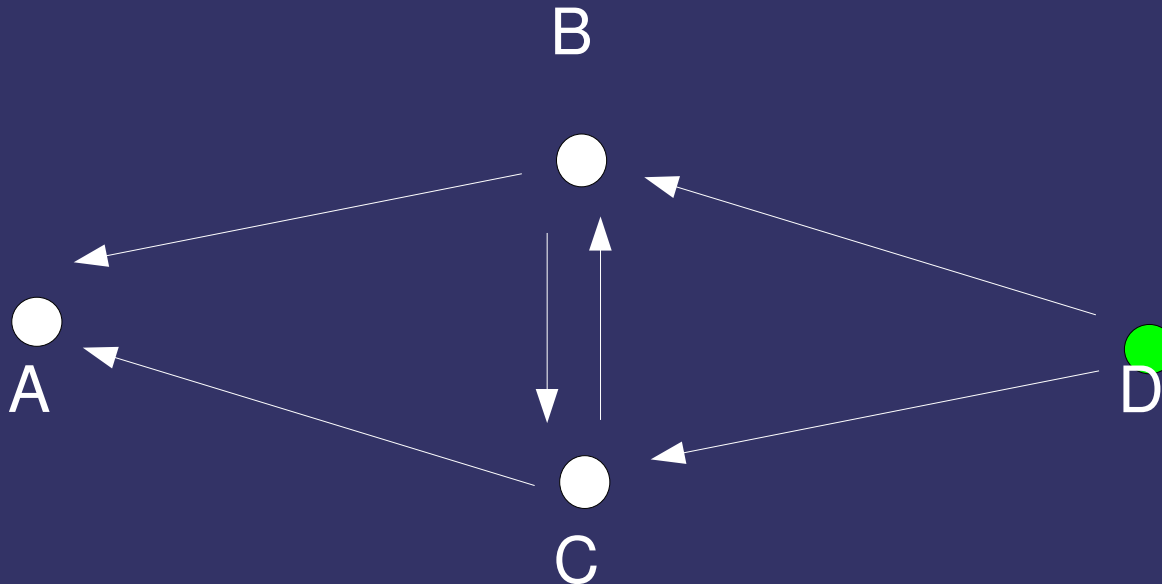
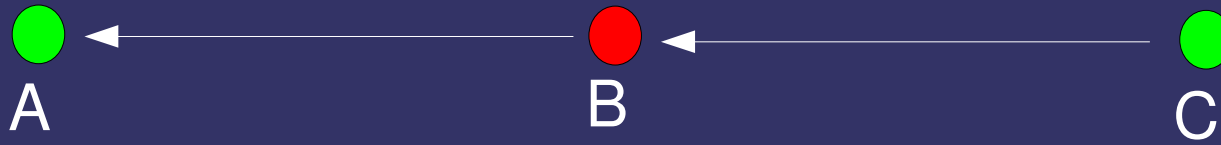
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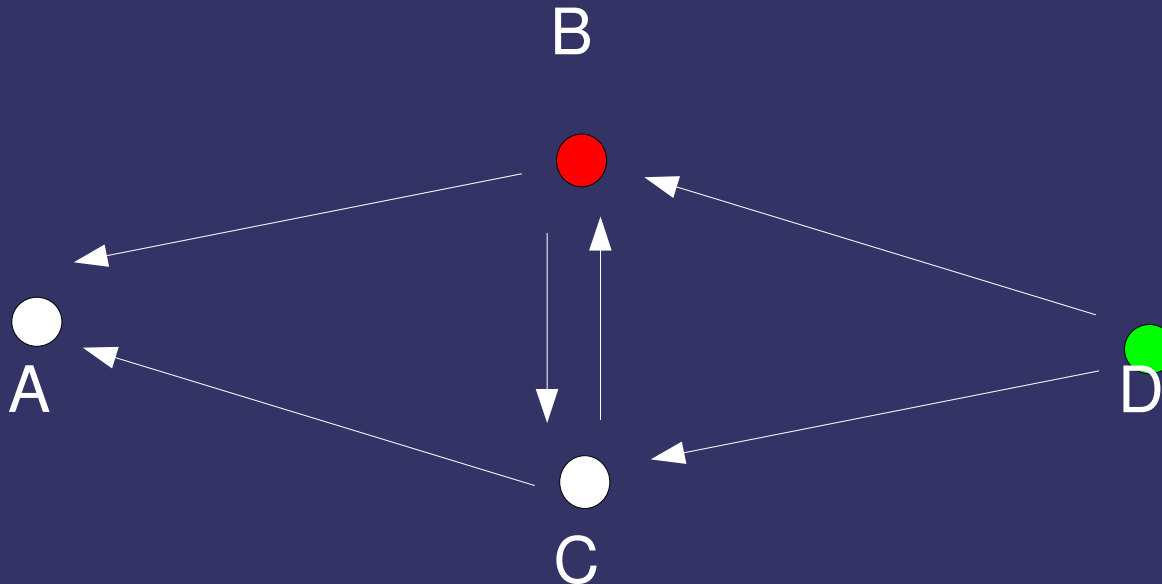
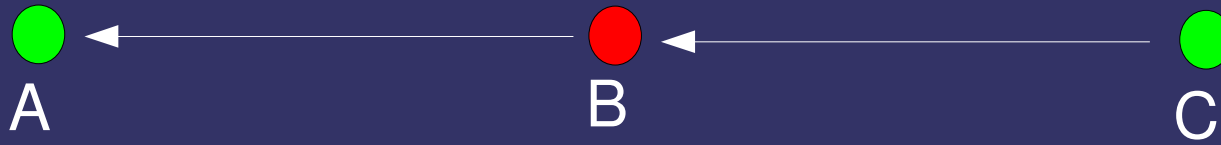
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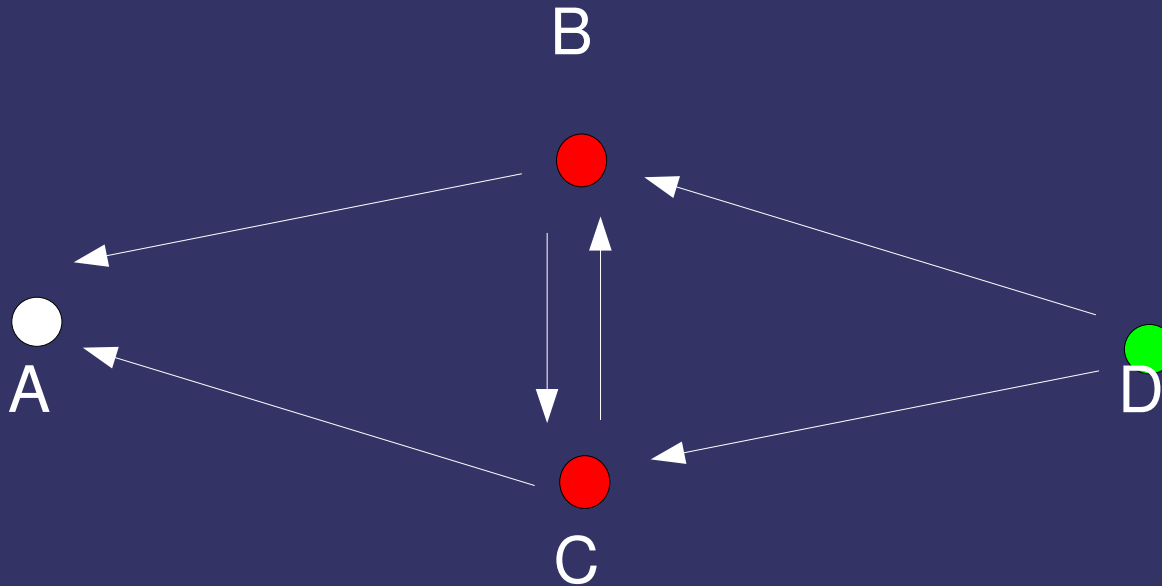
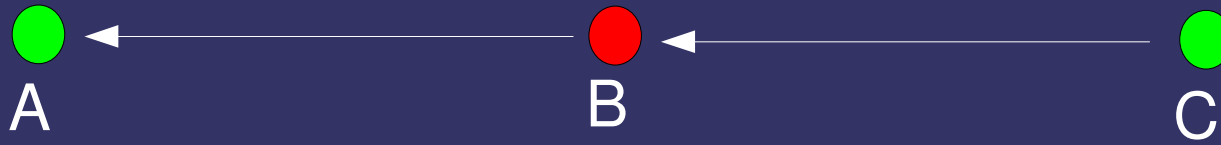
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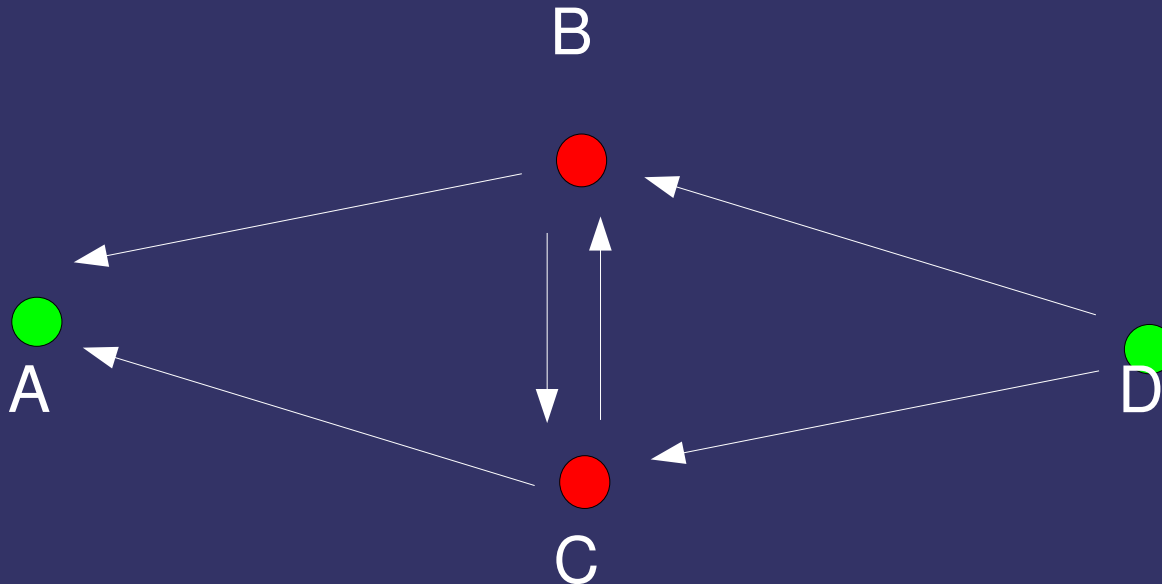
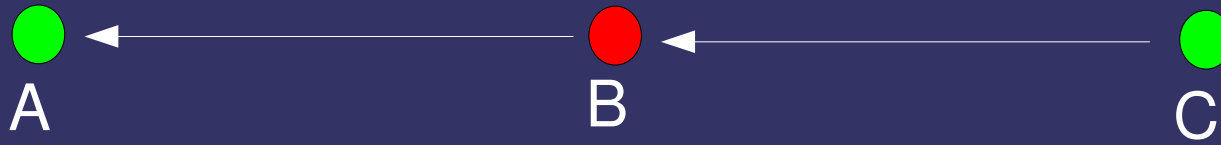
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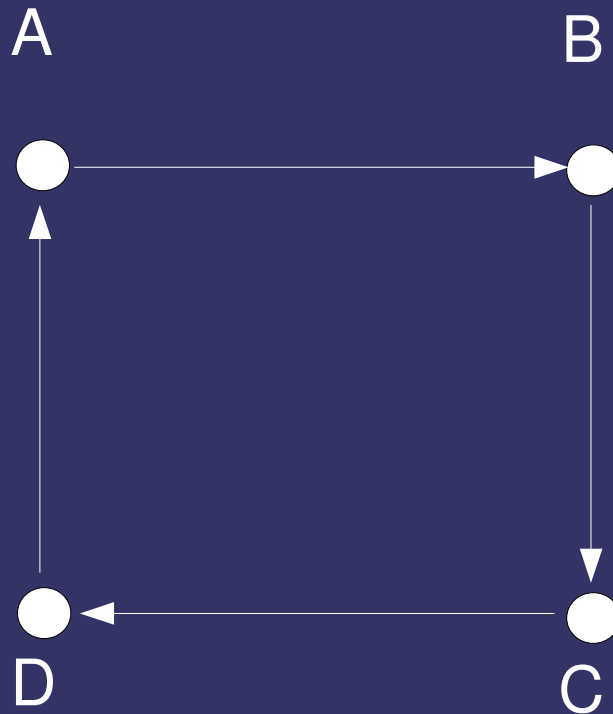
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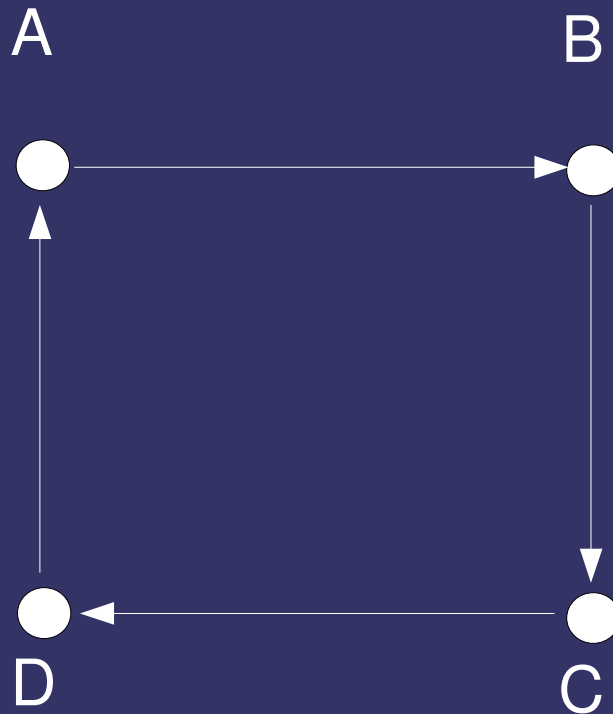


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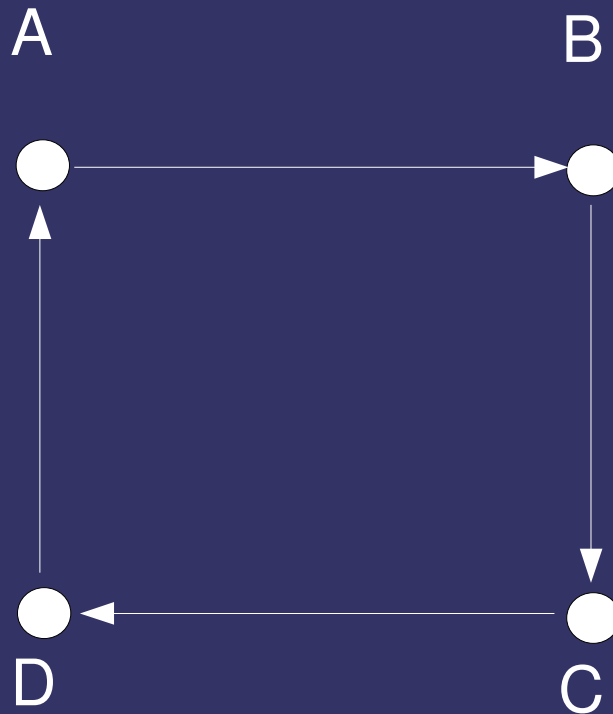




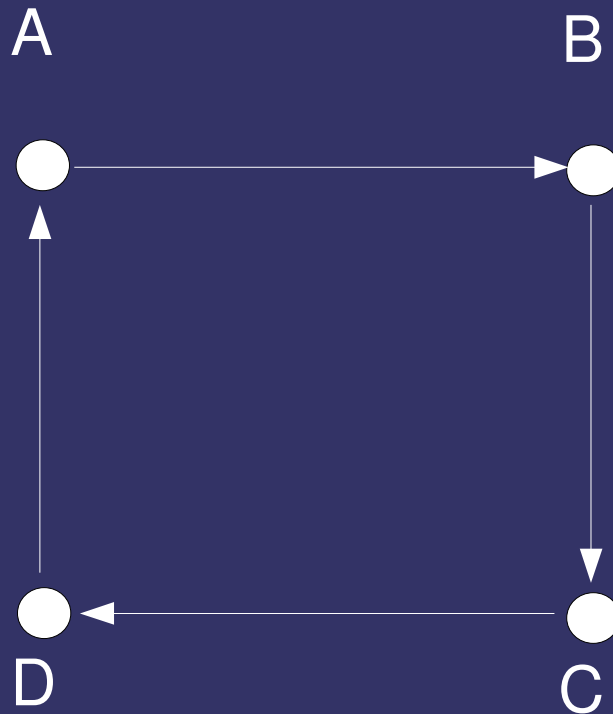
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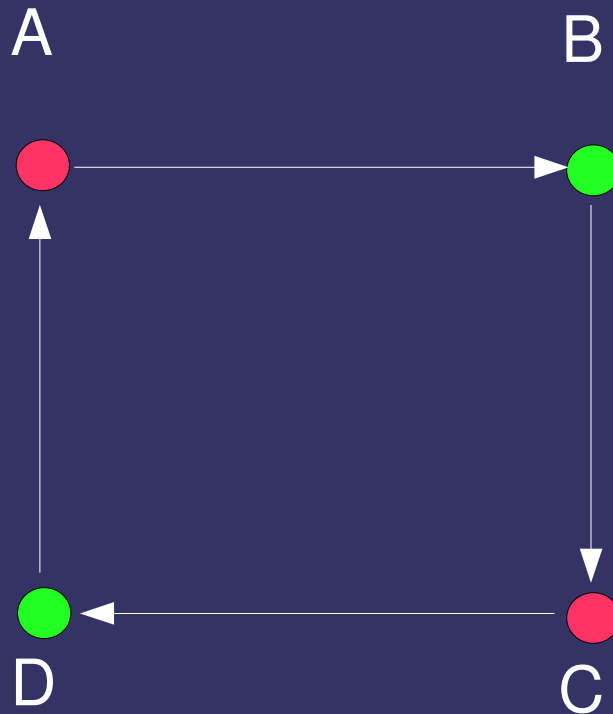
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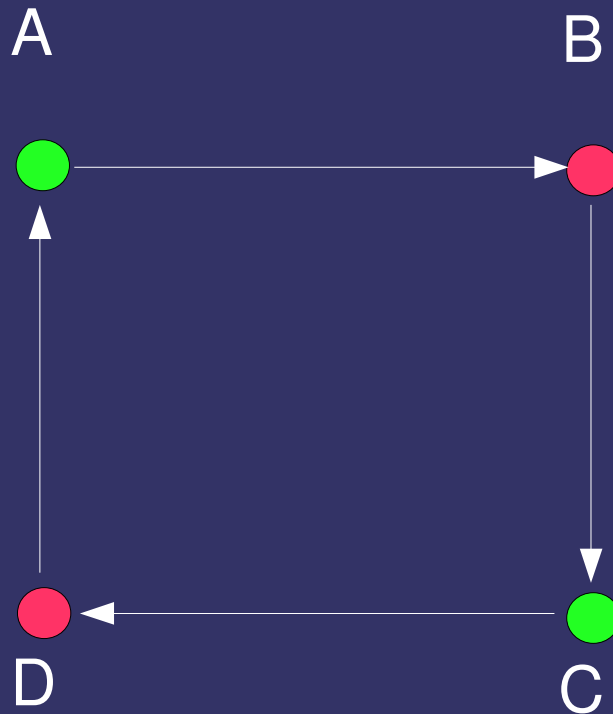
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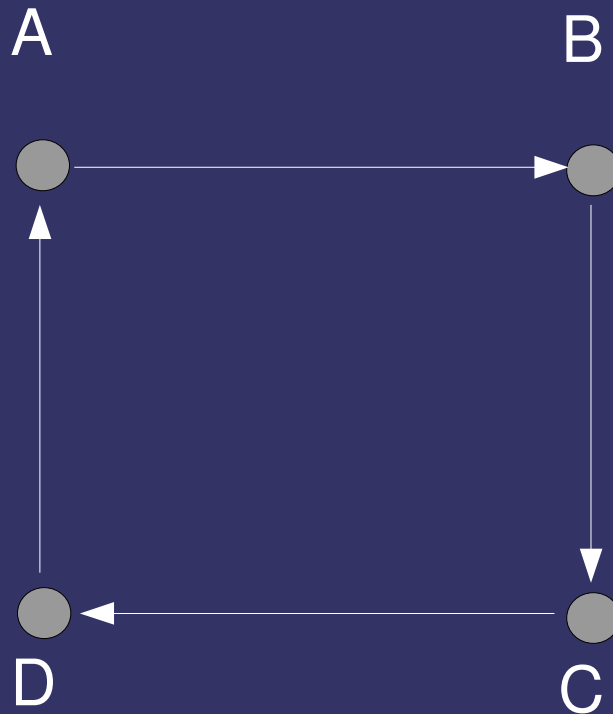
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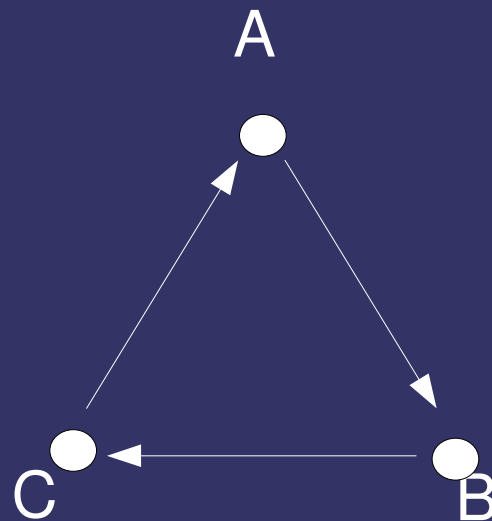
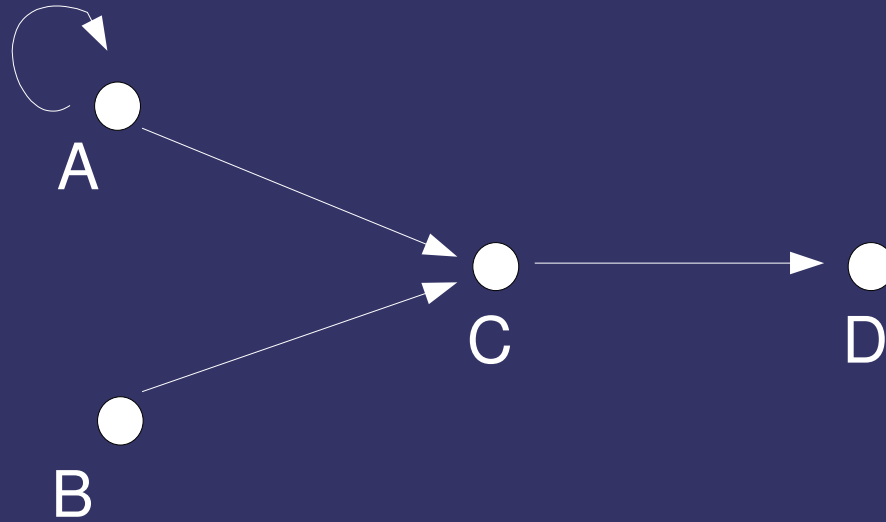
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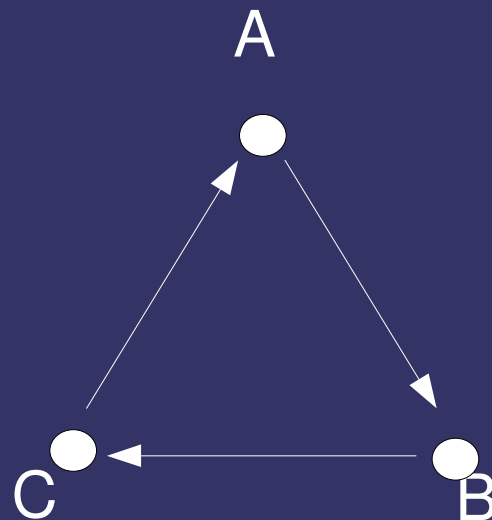
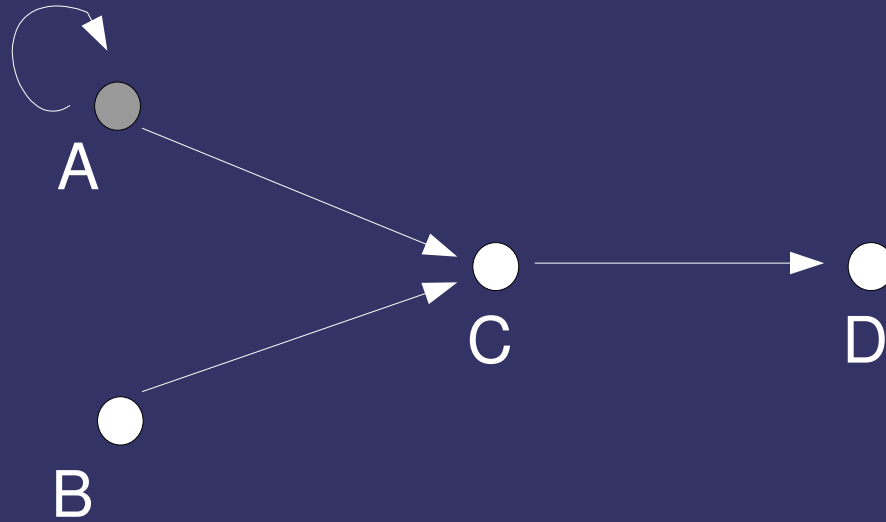
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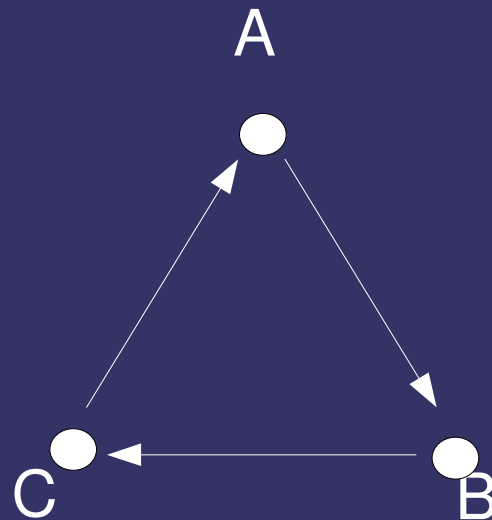
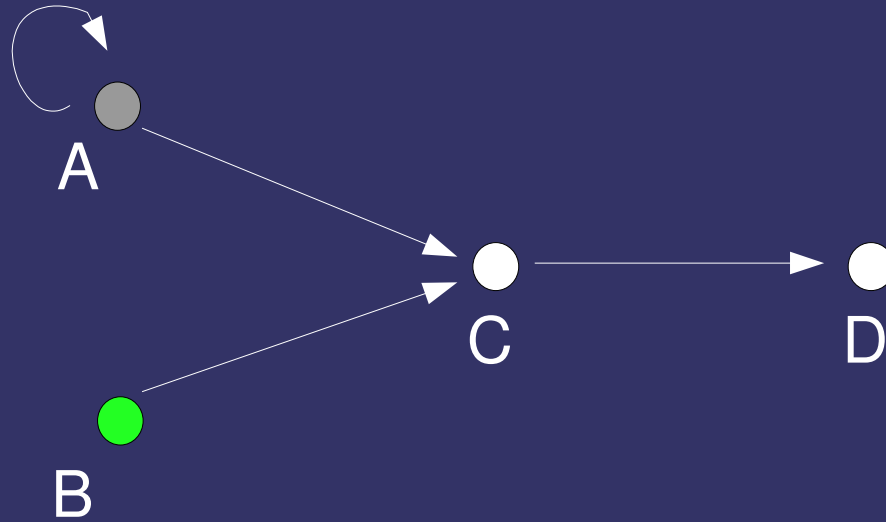


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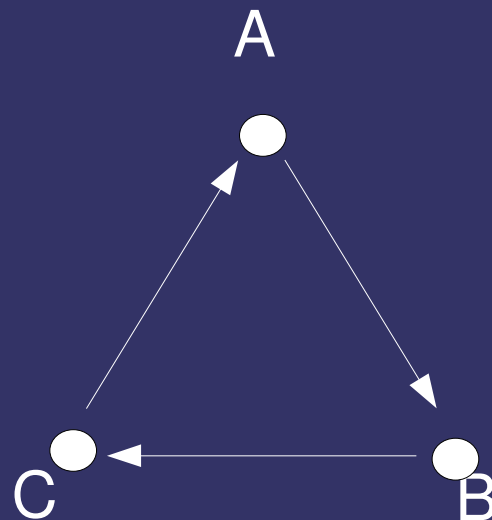
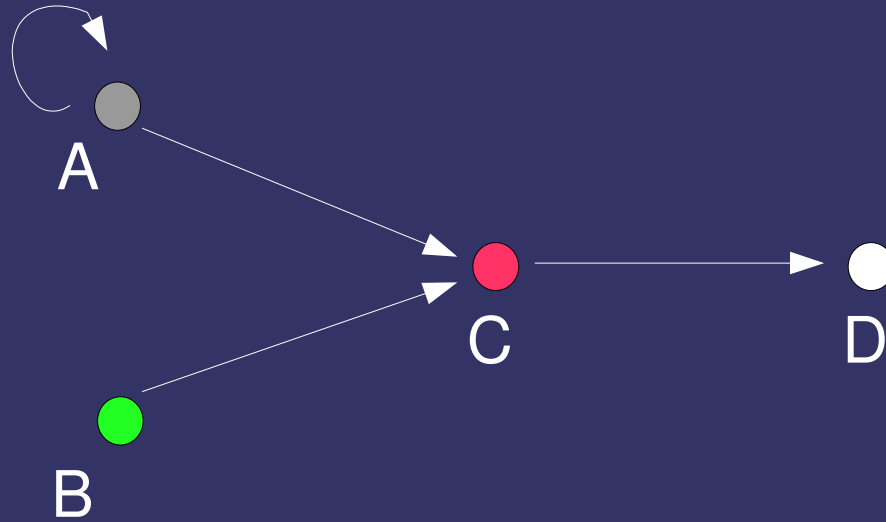




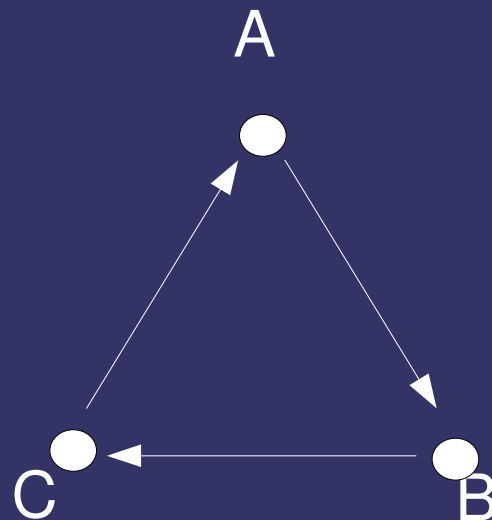
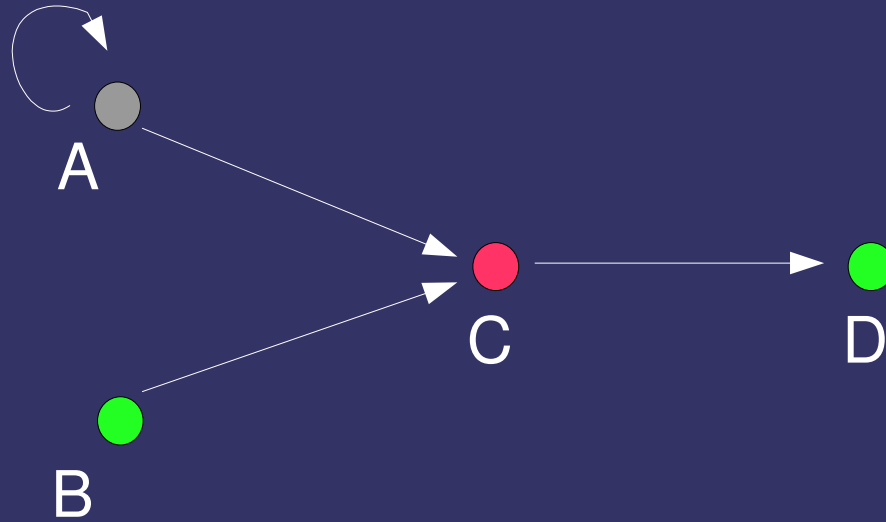
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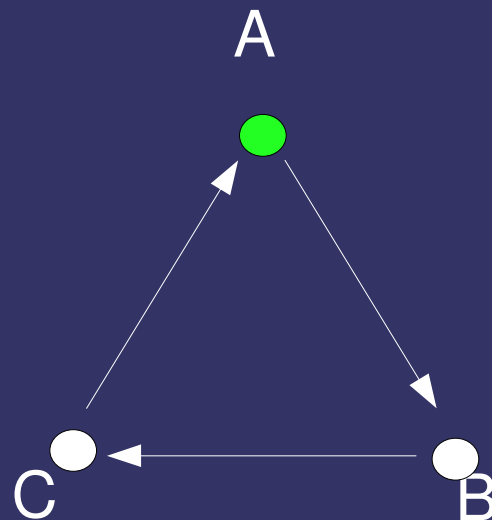
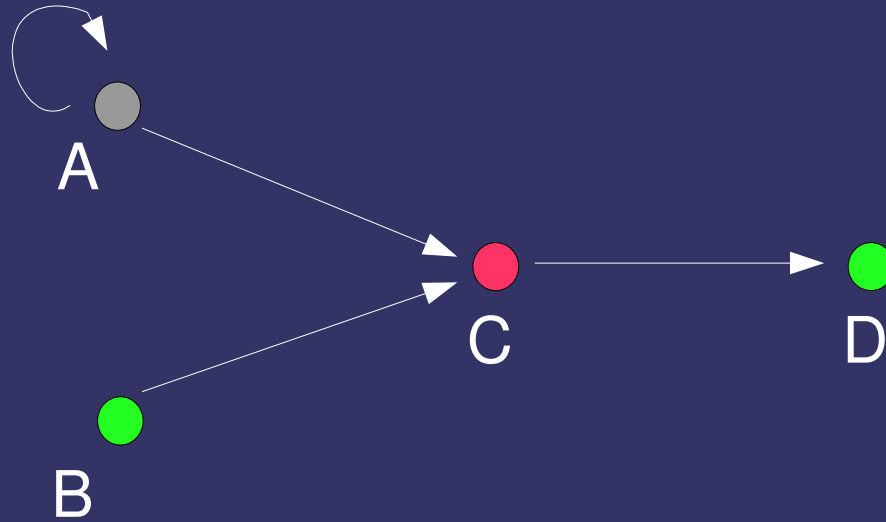
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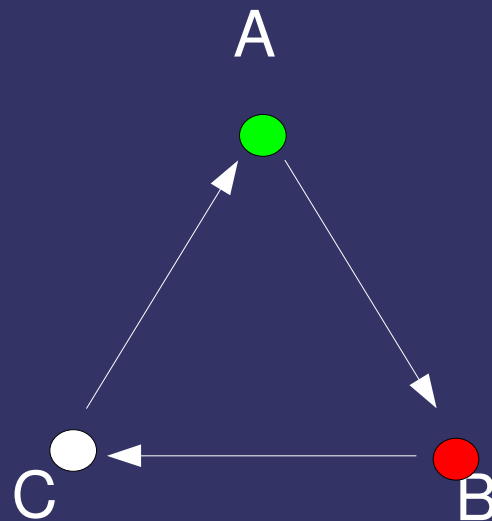
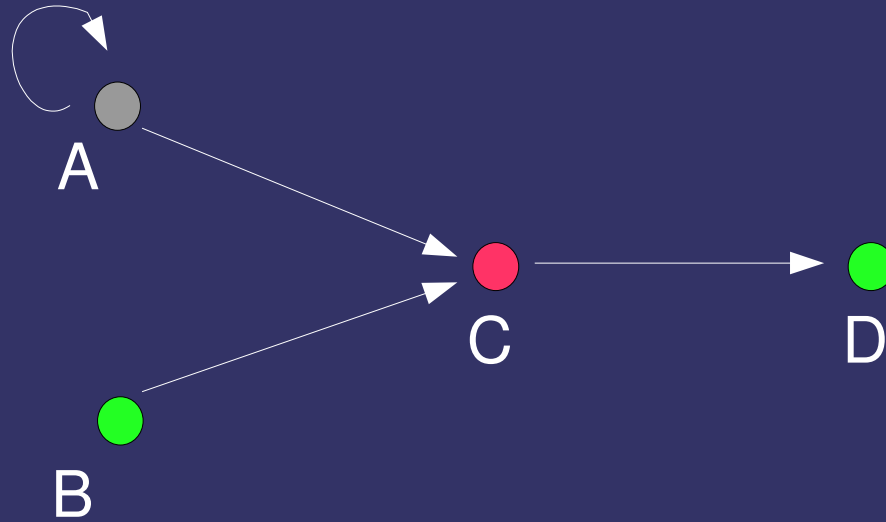
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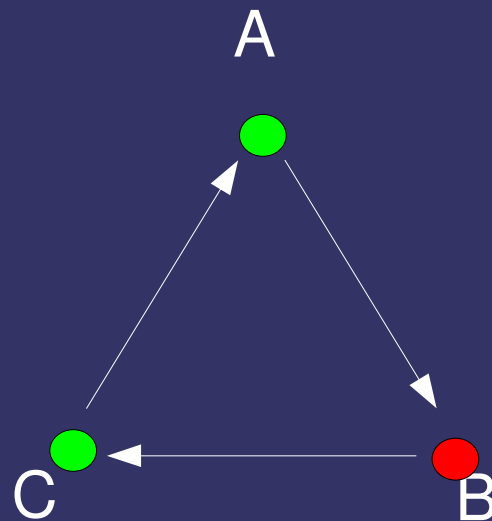
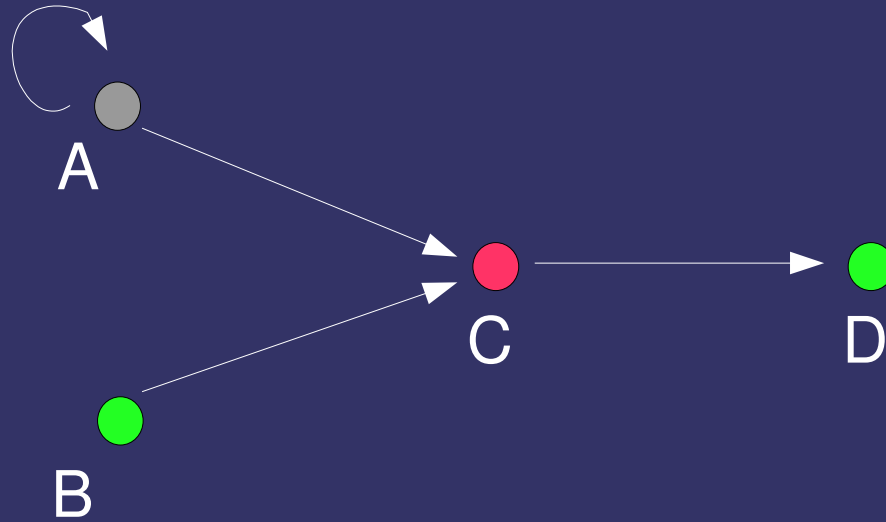
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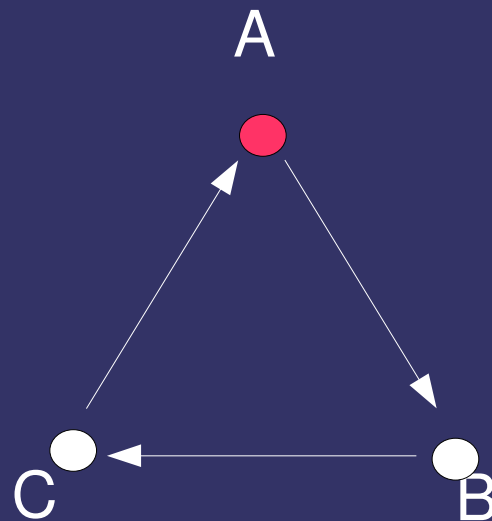
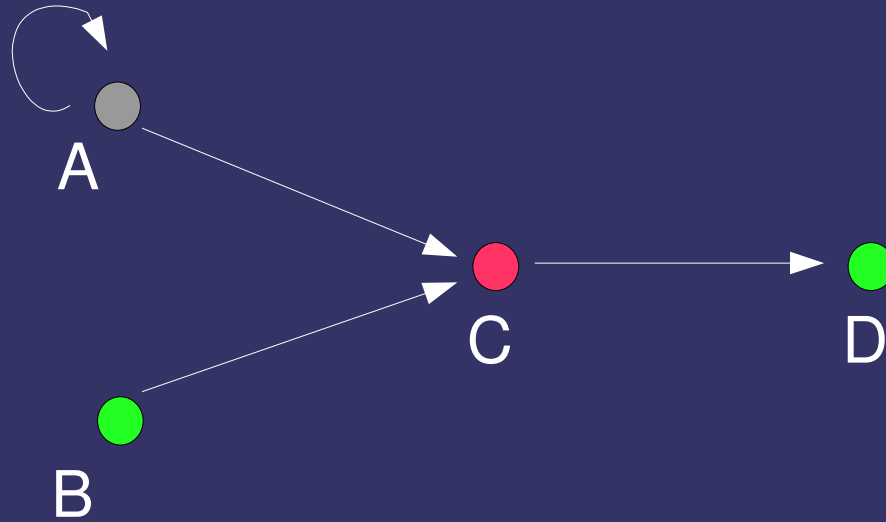
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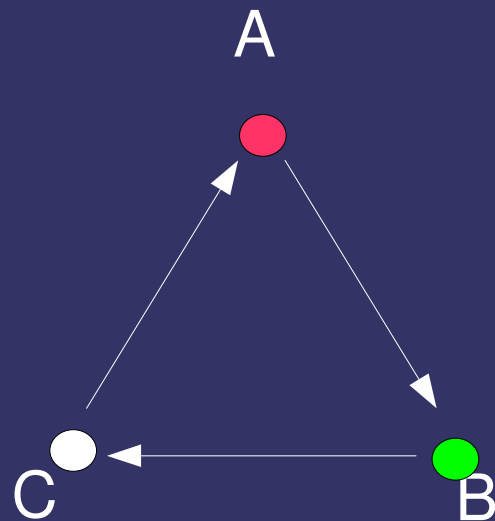
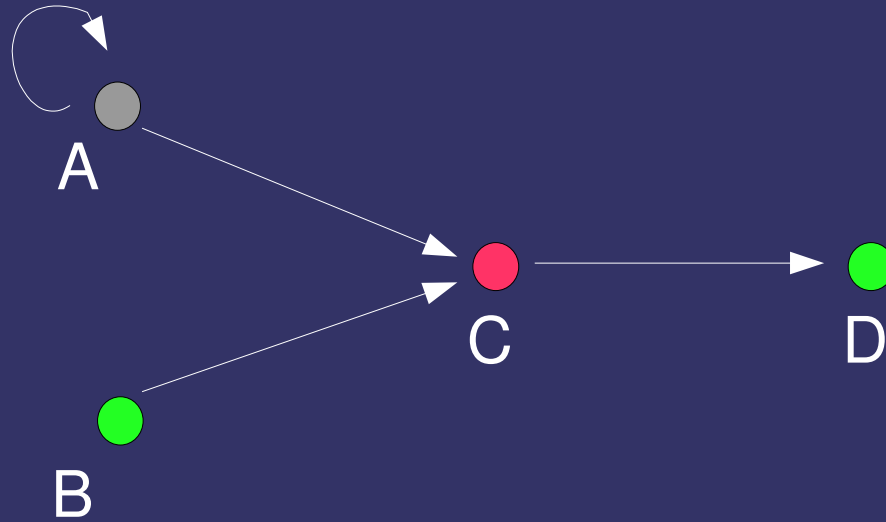
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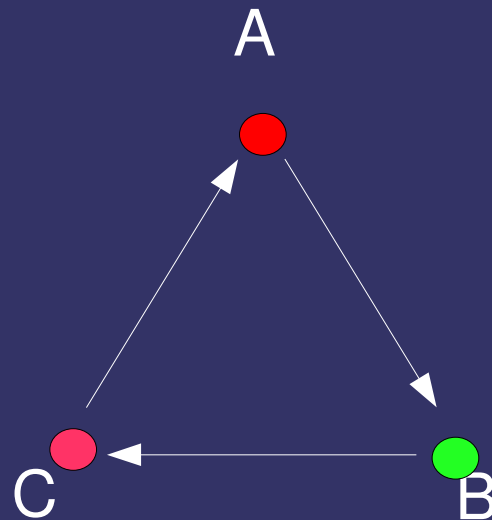
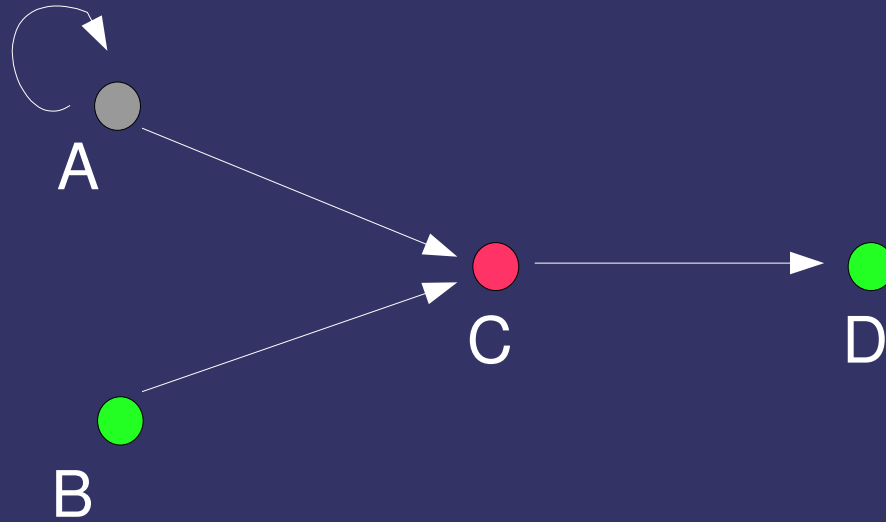


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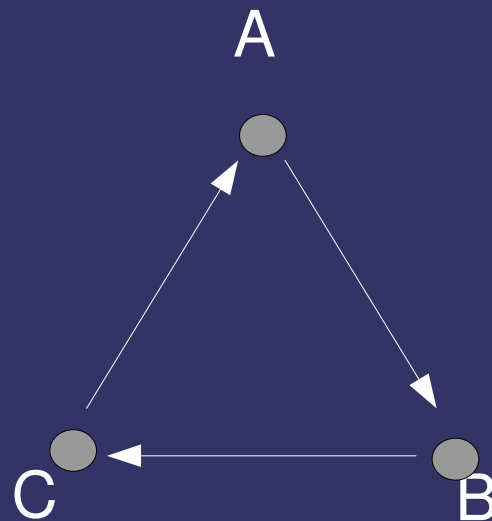
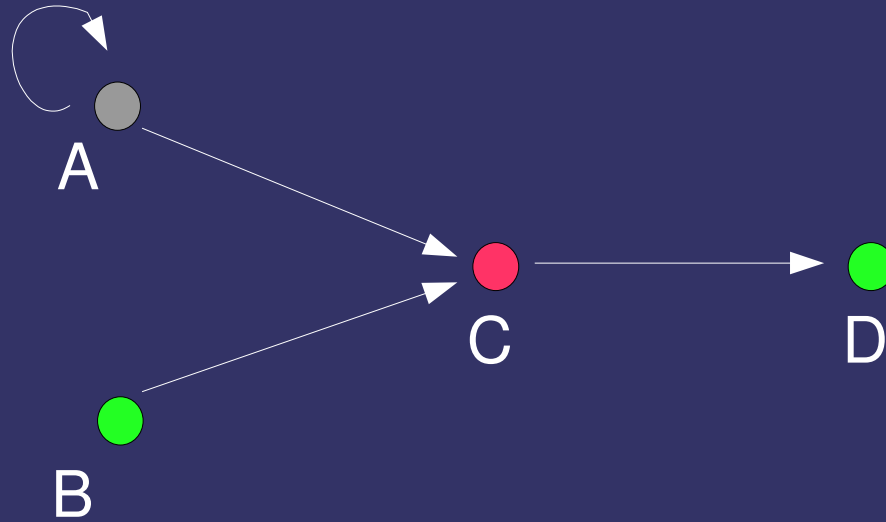




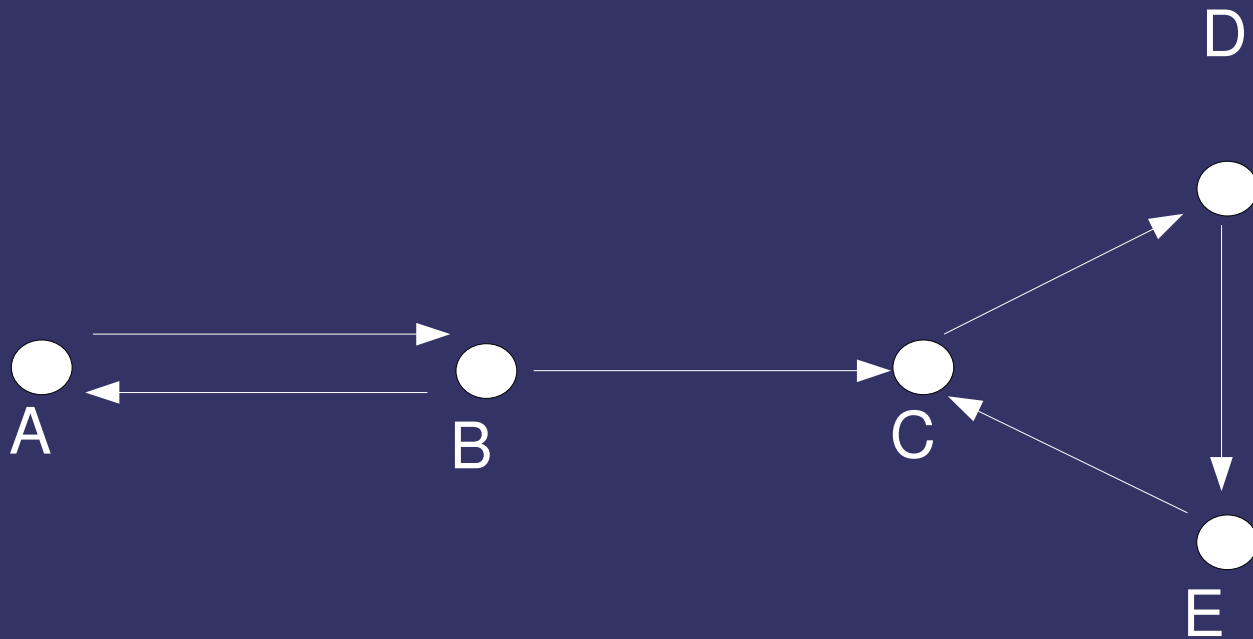
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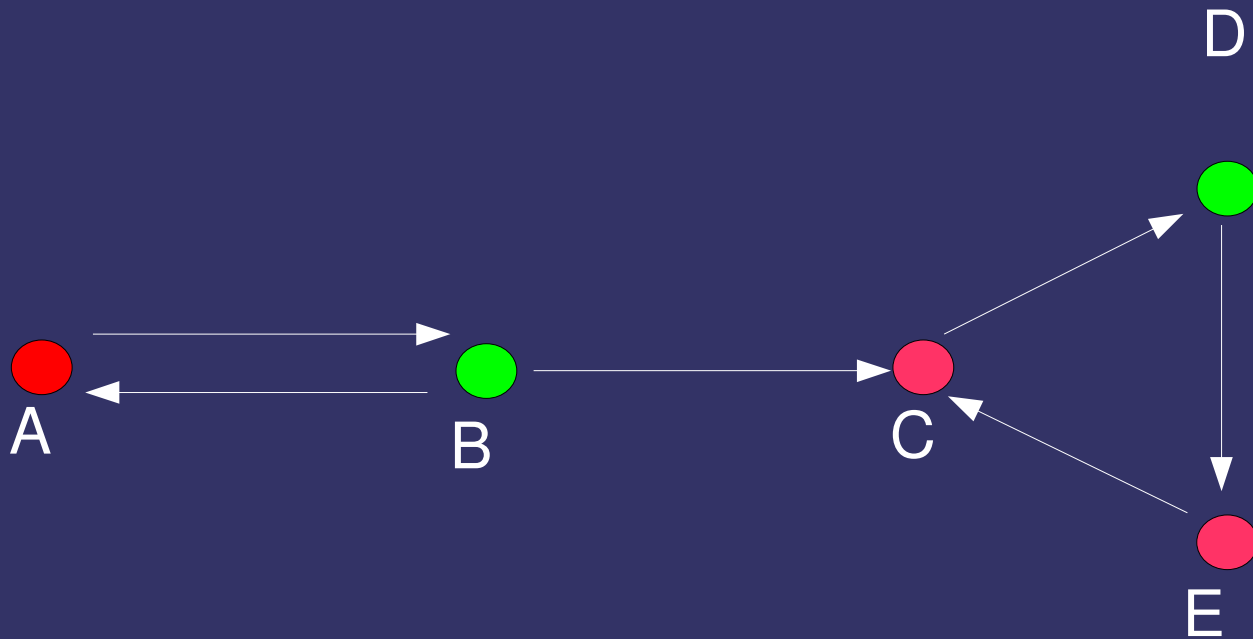


# ***Exercise 1***



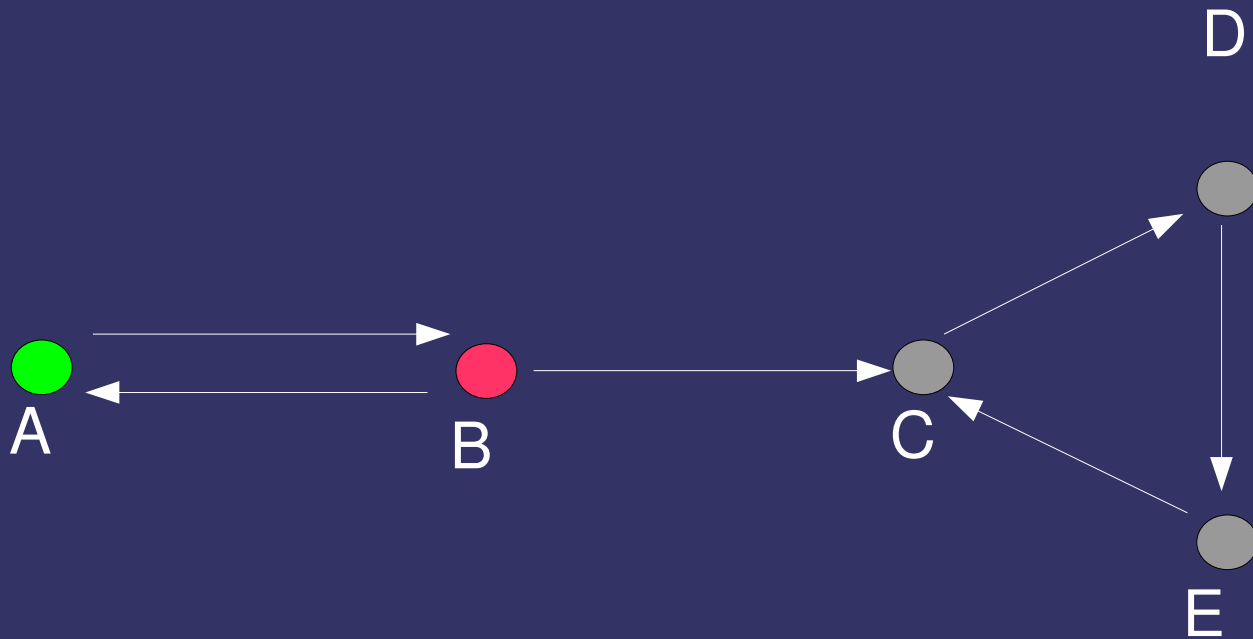
Give the three labellings of this argumentation framework

# Exercise 1



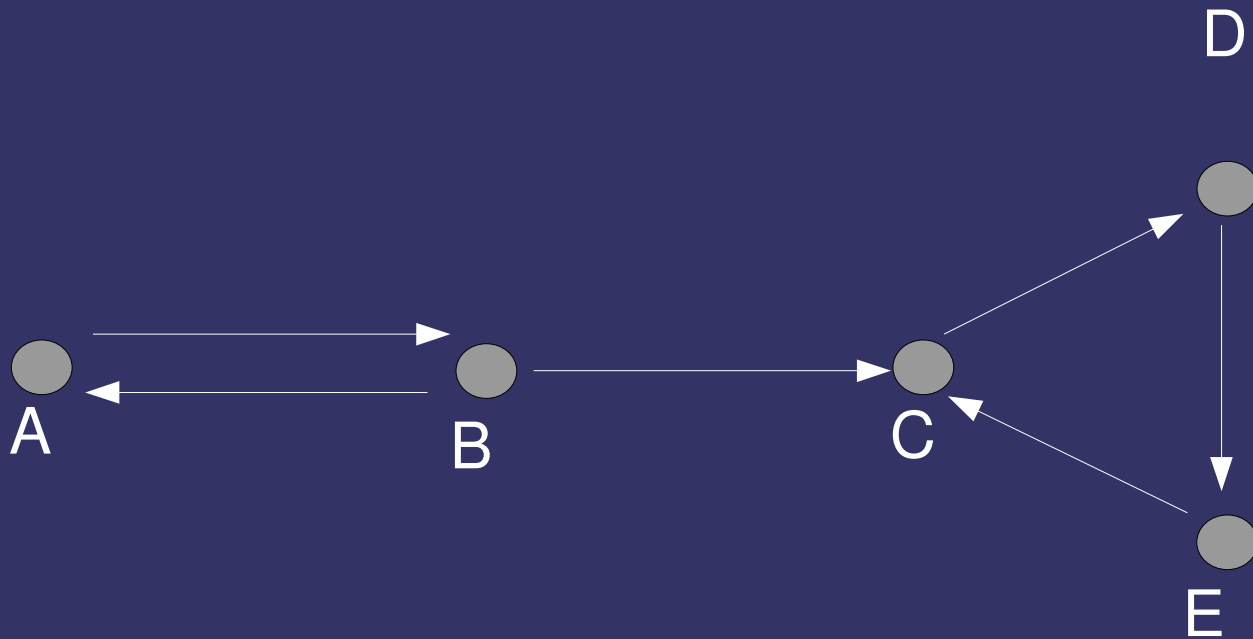
Give the three labellings of this argumentation framework

# Exercise 1



Give the three labellings of this argumentation framework

# ***Exercise 1***



Give the three labellings of this argumentation framework

# ***Argument Evaluation in the Literature (1/3)***

Args is conflict-free iff

Args does not contain A,B such that A defeats B

Args defends an argument A iff

for each argument B that defeats A,

Args contains an argument (C) that defeats B

$F(\text{Args})$  = all arguments defended by Args

# ***Argument Evaluation in the Literature (2/3)***

A conflict-free set of arguments  $\text{Args}$  is called:  
admissible iff  $\text{Args} \subseteq F(\text{Args})$

a complete extension iff  
 $\text{Args} = F(\text{Args})$

a grounded extension iff

$\text{Args}$  is the minimal complete extension

a preferred extension iff

$\text{Args}$  is a maximal admissible set

a stable extension iff  $\text{Args}$  is a conflict-free  
set that defeats everything not in it

a semi-stable extension iff  $\text{Args}$  is an  
admissible set with  $\text{Args} \cup \text{Args}^+$  maximal



# ***Argument Evaluation in the Literature (3/3)***

A conflict-free set of arguments  $\text{Args}$  is called:

admissible iff  $\text{Args} \subseteq F(\text{Args})$

a complete extension iff

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a grounded extension iff

$\text{Args}$  is the minimal complete extension

a preferred extension iff

$\text{Args}$  is a maximal complete extension

a stable extension iff  $\text{Args}$  is a complete extension that defeats everything not in it

a semi-stable extension iff  $\text{Args}$  is a

complete extension with  $\text{Args} \cup \text{Args}^+$  maximal

# ***Literature and Labellings***

## ***restriction on compl. labeling***

no restrictions

empty undec

maximal in

maximal out

maximal undec

minimal in

minimal out

minimal undec

## ***Dung-style semantics***

complete semantics

stable semantics

preferred semantics

preferred semantics

grounded semantics

grounded semantics

grounded semantics

semi-stable semantics

# ***Some properties of argument semantics***

grounded extension =  $\cap$  complete extensions  
[Dung 1995 AIJ]

an argument is in at least one preferred extension  
iff it is in at least one complete extension  
iff it is in at least one admissible set.

# ***Computing the Grounded Extension***

Idea: start with the undefeated arguments,  
then iteratively add the defended arguments

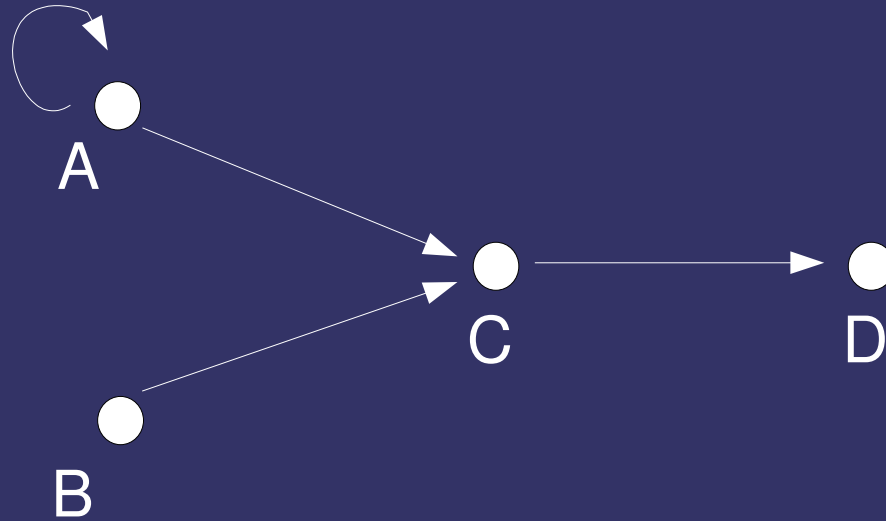
$$F^0 = \emptyset$$

$$F^{i+1} = \{ A \mid A \text{ is defended by } F^i \}$$

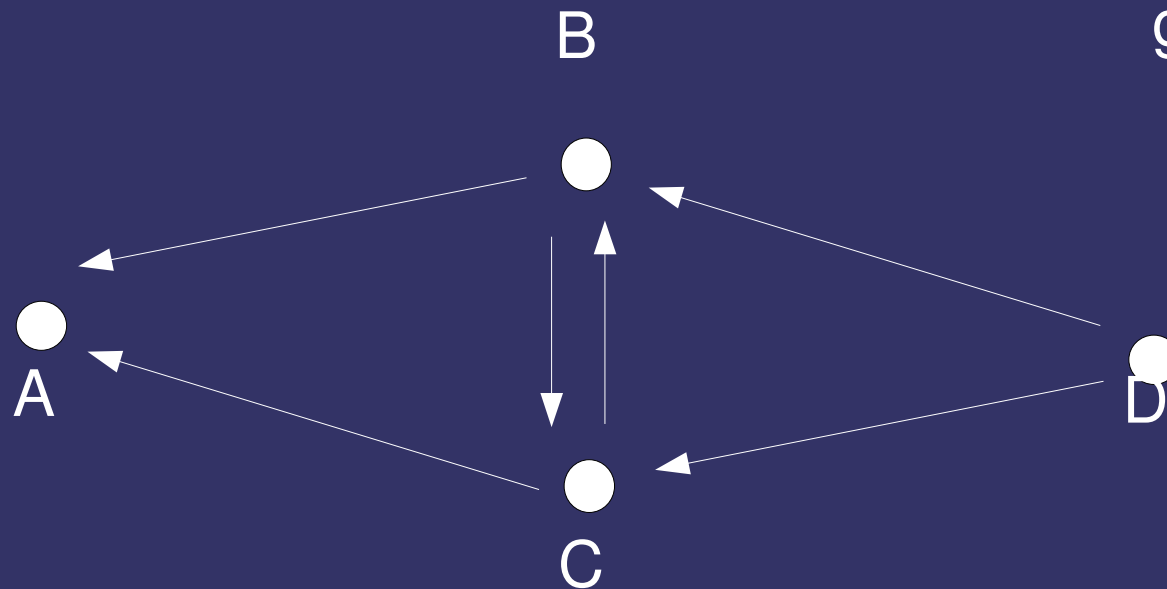
$$F^\infty = \bigcup_{i=0 \dots \infty} F^i$$

If each argument has a finite set of defeaters,  
then  $F^\infty$  is the grounded extension.

## Exercise 2



Give for each of these argumentation frameworks the grounded extension



# ***A Dialectical Game for Grounded Semantics***

Is argument A element of the grounded extension?

proponent states A

opponent and proponent then take turns, in which they  
state an argument that defeats the previous argument

proponent is not allowed to repeat any previous argument

a player wins iff the other player cannot move

Argument A is in the grounded extension iff

proponent has winning strategy for A

# ***A Dialectical Game for Admissibility***

Is argument A element of an admissible set?

proponent states A

opponent and proponent then take turns;

the opponent each time states an argument that defeats *one of the previous arguments* of the proponent; the proponent each time states an argument that defeats *the immediately preceding argument* of the opponent

the proponent may repeat its own moves, but not the moves of the opponent;

the opponent may repeat the proponent's moves but not its own moves

proponent wins iff opponent cannot move;

opponent wins iff proponent cannot move or if opponent is able to repeat proponent's move

A is in admissible set iff proponent can win game

# ***Default Logic as Argumentation***

default:  $\text{pre}(d) : \text{jus}(d) / \text{cons}(d)$

arguments of the form:  $(d_1, \dots, d_n)$  where

for each  $d_i$  ( $1 \leq i \leq n$ ) it holds that

$\{\text{cons}(d_1), \dots, \text{cons}(d_{i-1})\} \cup W \vdash \text{pre}(d_i)$

$(d_1, \dots, d_n)$  defeats  $(d'_1, \dots, d'_m)$  iff

there is some  $d'_i$  ( $1 \leq i \leq m$ ) such that

$\{\text{cons}(d_1), \dots, \text{cons}(d_n)\} \cup W \vdash \neg \text{jus}(d'_i)$

stable semantics



# ***Pollock***

Arguments of the form  $(\text{pfrule}_1, \dots, \text{pfrule}_n)$  where  
 $\{\text{cons}(\text{pfrule}_1), \dots, \text{cons}(\text{pfrule}_{i-1})\} \cup W \vdash \text{ant}(\text{pfrule}_i)$

defeat: rebutting + undercutting

preferred semantics (before: grounded semantics)

# ***Logic Programming***

Arguments: trees constructed with rules. The children of a rule

$c \leftarrow a_1, \dots, a_n, \text{not } b_1, \dots, \text{not } b_m$

are rules with heads  $a_1, \dots, a_n$

An argument  $A$  defeats an argument  $B$  iff

$A$  contains a rule with  $c$  as its head and

$B$  contains a rule with  $\text{not } c$  in its body

stable semantics (“stable model semantics”)

grounded semantics (“well-founded semantics”)

# ***How Things Go Wrong (1/5)***

$r$	$r \Rightarrow m$	$m \rightarrow hs$
$p$	$p \Rightarrow b$	$b \rightarrow \neg hs$

$A1 = (r) \Rightarrow m$

$A2 = (p) \Rightarrow b$

$A3 = A1 \rightarrow hs$

$A4 = A2 \rightarrow \neg hs$

Conclusions  $m$  and  $b$  are justified under any semantics but what about  $hs$  and  $\neg hs$ ?

# *How Things Go Wrong (2/5)*

$r$	$r \Rightarrow m$	$m \supset hs$	$(\rightarrow \equiv \vdash)$
$p$	$p \Rightarrow b$	$b \supset \neg hs$	

A1:  $(r) \Rightarrow m$

A2:  $(p) \Rightarrow b$

A3:  $(A1, m \supset hs) \rightarrow hs$

A4:  $(A2, b \supset \neg hs) \rightarrow \neg hs$

A5:  $(A3, b \supset \neg hs) \rightarrow \neg b$

A6:  $(A4, m \supset hs) \rightarrow \neg m$

So far,  
so good...

# *How Things Go Wrong (3/5)*

$j \quad j \Rightarrow s$

$(\text{"}\rightarrow\text{"} \equiv \text{"}\vdash\text{"}) \quad m \quad m \Rightarrow \neg s \quad wf \quad wf \Rightarrow r$

There now exist the following arguments:

$A = (j) \Rightarrow s$

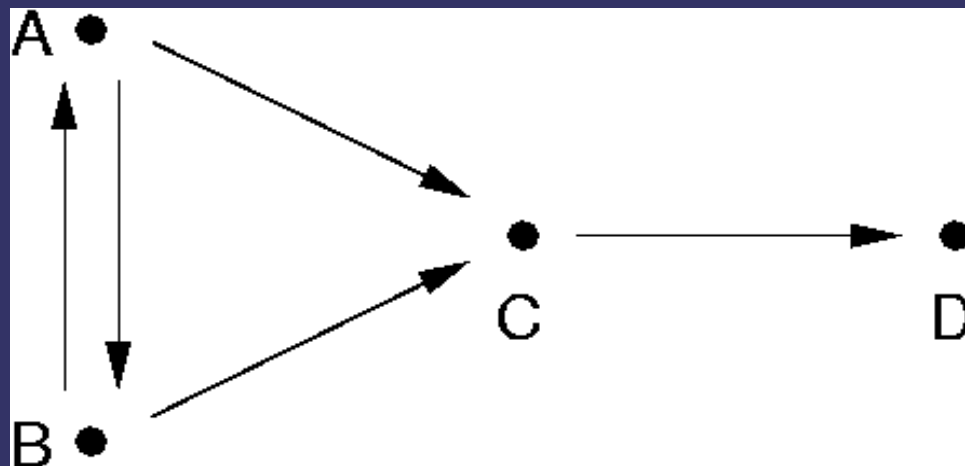
$B = (m) \Rightarrow \neg s$

$D = (wf) \Rightarrow r$

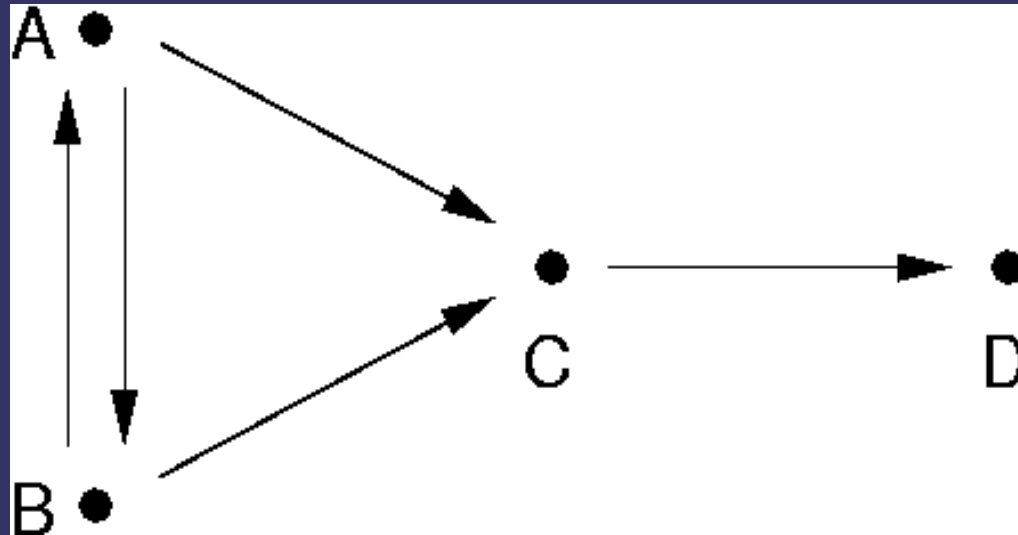
(unfortunately,

there also exists:

$C = A, B \rightarrow \neg r$ )



# *How Things Go Wrong (4/5)*



Grounded semantics: no justified arguments  
Why not use preferred or stable semantics?  
Reiter and Pollock also do this...

# *How Things Go Wrong (5/5)*

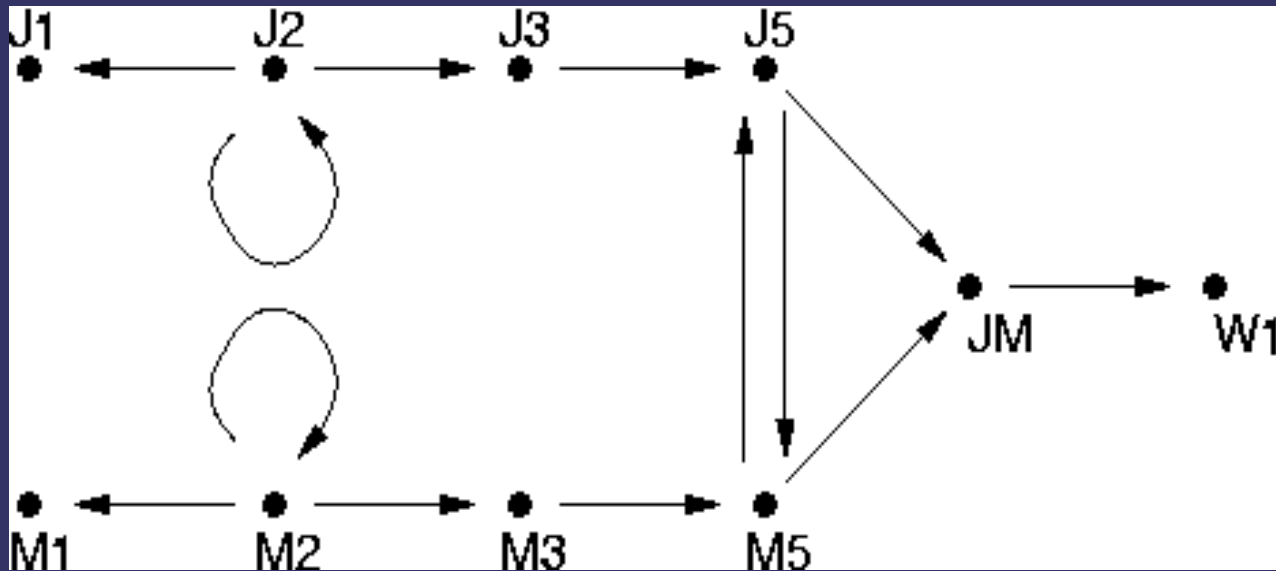
John: “Cup of coffee contains sugar.”

Mary: “Cup of coffee doesn't contain sugar.”

John: “I'm unreliable.”

Mary: “I'm unreliable.”

Weather Forecaster: “Tomorrow rain.”



# ***Quality Postulates***

Let  $J$  be the justified conclusions and  
 $Cl_S(J)$  be the closure of  $J$  under the rules in  $S$ .

direct consistency:  $\neg \exists p: (p \in J \wedge \neg p \in J)$

closedness:  $J = Cl_S(J)$

indirect consistency:  $Cl_S(J)$  is consistent

crash-resistancy:

“Local problems should not have global effects”

non-interference:

“A set of formulas should not be able to influence the entailment of a totally unrelated set of formulas, when being merged to it”



# *Transposition*

Let  $s$  be a strict rule of the form

$$a_1, \dots, a_n \rightarrow c$$

A rule  $s'$  is a *transposition* of  $s$  iff  $s'$  is of the form

$$a_1, \dots, a_{i-1}, \neg c, a_{i+1}, \dots, a_n \rightarrow \neg a_i$$

(for some  $1 \leq i \leq n$ )

A set of strict rules  $S$  is closed under transposition iff for each  $s \in S$ , if  $s'$  is a transposition of  $s$  then  $s' \in S$ .

# ***Restricted versus unrestricted rebut***

$$((a) \Rightarrow b) \Rightarrow c$$

$$((d) \Rightarrow e) \Rightarrow \neg c$$

# ***Restricted versus unrestricted rebut***

$$((a) \rightarrow b) \rightarrow c$$

$$((d) \Rightarrow e) \Rightarrow \neg c$$

# ***Restricted versus unrestricted rebut***

$$((a) \Rightarrow b) \rightarrow c$$

$$((d) \rightarrow e) \Rightarrow \neg c$$

# ***Restricted versus unrestricted rebut***

$((a) \Rightarrow b) \rightarrow c$

$((d) \rightarrow e) \Rightarrow \neg c$

*unrestricted rebut:*

an argument can be rebutted on a conclusion derived by at least one defeasible rule

*restricted rebut:*

an argument can be rebutted only on the direct consequent of a defeasible rule

# ***Satisfying the quality postulates***

Two possibilities:  
strict rules closed under transposition  
+ unrestricted rebut  
+ grounded semantics  
strict rules closed under transposition  
+ restricted rebut  
+ any “well behaved” semantics

With “well behaved” semantics we mean a semantics that yields a non-empty subset of the complete extensions (e.g. preferred, grounded, complete, ideal, semi-stable, ...)

# ***Floating conclusions (1/2)***

“Lars has a Dutch mother, so he's probably Dutch, so he probably likes ice-skating”

“Lars has a Norwegian father, so he's probably Norwegian, so he probably likes ice-skating”

A:  $((\text{dutch\_mom}) \Rightarrow \text{dutch}) \Rightarrow \text{likes\_skating}$

B:  $((\text{norw\_dad}) \Rightarrow \text{norw}) \Rightarrow \text{likes\_skating}$

C:  $((\text{dutch\_mom}) \Rightarrow \text{dutch}) \rightarrow \neg \text{norw}$

D:  $((\text{norw\_dad}) \Rightarrow \text{norw}) \rightarrow \neg \text{dutch}$

Here, C defeats A and D, and D defeats C and B.

Grounded extension:  $\emptyset$

Wanted: *floating conclusions*

## ***Floating conclusions (2/2)***

“Witness X says the suspect killed the victim with an axe on Monday morning”

“Witness Y says the suspect killed the victim with a rifle on Monday afternoon”

A:  $((\text{decl\_X}) \Rightarrow \text{story\_X}) \Rightarrow \text{guilty}$

B:  $((\text{decl\_Y}) \Rightarrow \text{story\_Y}) \Rightarrow \text{guilty}$

C:  $((\text{decl\_X}) \Rightarrow \text{story\_X}) \rightarrow \neg \text{story\_Y}$

D:  $((\text{decl\_Y}) \Rightarrow \text{story\_Y}) \rightarrow \neg \text{story\_X}$

Here, C defeats A and D, and D defeats C and B.

Grounded extension:  $\emptyset$

Now, do we still want floating conclusions?



# ***Non-admissibility based semantics (1/2)***

Why not weaken the requirement of admissibility to, for instance, just conflict-freeness?

For example, why not define an extension as a set  $Args$  with maximal range ( $Args \cup Args^+$ )

Advantage: it treats even and odd loops in the same way.

# ***Non-admissibility based semantics (2/2)***

Suppose we have the following non-defeasible information:  $\{ a, b, \neg(c \wedge d) \}$  as well as

two defeasible rules:  $a \Rightarrow c$  and  $b \Rightarrow d$

Let the strict rules be based on classical entailment

A:  $(a) \Rightarrow c$

B:  $(b) \Rightarrow d$

C:  $((a) \Rightarrow c), \neg(c \wedge d) \rightarrow \neg d$

D:  $((b) \Rightarrow d), \neg(c \wedge d) \rightarrow \neg c$

E:  $\neg(c \wedge d)$

$\{A, B, E\}$  is conflict-free,

even though it yields inconsistent conclusions!

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